Pre-Calculus CP 1 – Section 8.2 Notes **Name:** KEY
Matrix Operations

A **Matrix** is an arrangement of numbers in rows and columns rows by columns

The **Dimensions** of a matrix are written as m x n, where m is rows and n is columns

The **Entries** of a matrix are the numbers within the brackets

Examples: Give the dimensions of the matrices below:

$3\left\{\text{A) } \overbrace{\begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 0 & -1 & -4 \end{bmatrix}}^{3}\right.$ 3x3 $\quad 2\left\{\text{B) } \overbrace{\begin{bmatrix} 1 & 0 & 3 \\ -5 & 9 & 7 \end{bmatrix}}^{3}\right.$ 2x3 $\quad 4\left\{\text{B) } \overbrace{\begin{bmatrix} 0 \\ -4 \\ -7 \\ 2 \end{bmatrix}}^{1}\right.$ 4x1

In matrix A, the entry in the third row, second column is written a_{32} and is equal to -1

Two matrices are **equal** if they have the same numbers in the same entries

Examples: Are the following matrices equal?

a) $\begin{bmatrix} 5 & 0 \\ -\frac{3}{4} & \frac{4}{4} \end{bmatrix} \quad \begin{bmatrix} 5 & 0 \\ -.75 & 1 \end{bmatrix}$ yes!

b) $\begin{bmatrix} 2 & 6 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} -2 & 6 \\ -3 & 0 \end{bmatrix}$ NO

To ADD or SUBTRACT matrices, you add or subtract corresponding entries

*** You can ONLY add or subtract matrices if they have the SAME DIMENSIONS***

Examples: Perform the indicated operation, if possible

a) $\begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ -9 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ 11 \end{bmatrix}$

b) $\begin{bmatrix} 8 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} 2 & -7 \\ 6 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ -2 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 9 & -7 \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ = DNE

d) $\begin{bmatrix} 7 & 8 & 1 \\ -9 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 7 \\ 1 & 3 & -3 \end{bmatrix} = \begin{bmatrix} 6 & 8 & -6 \\ -10 & -3 & 7 \end{bmatrix}$

e) $\begin{bmatrix} 3 & 6 \\ -3 & -6 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ -7 & 8 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ -10 & 11 \end{bmatrix} = \begin{bmatrix} -1 & 9 \\ 0 & -9 \end{bmatrix}$

In matrix algebra, a **real number** is often called a SCALAR

To multiply a matrix by a scalar, you multiply each entry in the matrix by the scalar.
This process is called **scalar multiplication**.

Examples: Perform the indicated operations:

a) $6\begin{bmatrix} 2 & -3 \\ 8 & 4 \end{bmatrix} = \begin{bmatrix} 12 & -18 \\ 48 & 24 \end{bmatrix}$

b) $-2\begin{bmatrix} 4 & -7 \\ 3 & 3 \\ 2 & -9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 9 & -8 \\ 1 & -4 \end{bmatrix} = \begin{bmatrix} -5 & 20 \\ 3 & -14 \\ -3 & 14 \end{bmatrix}$

When $k = -1$, the scalar product is $-A$ and is called the OPPOSITE matrix

If $A = \begin{bmatrix} 3 & -5 \\ 0 & 2 \end{bmatrix}$ then $-A = \begin{bmatrix} -3 & 5 \\ 0 & -2 \end{bmatrix}$ and if we added them the result would be $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(this is called the ZERO matrix)

You can also use matrices to set up equations and SOLVE for missing variables:

Ex.1) Solve $\begin{bmatrix} 2x \\ 2x+3y \end{bmatrix} = \begin{bmatrix} y \\ 12 \end{bmatrix}$

$2x = y$
$2x + 3y = 12$
$2x + 3(2x) = 12$
$8x = 12$
$x = \frac{12}{8} = \frac{3}{2}$
$y = 2\left(\frac{3}{2}\right) = 3$

Ex 2): solve the matrix equation for x and y

a_{12}

$4\left(\begin{bmatrix} 8 & 0 \\ -1 & 2y \end{bmatrix} + \begin{bmatrix} 4 & -2x \\ 1 & 6 \end{bmatrix}\right) = \begin{bmatrix} 48 & -48 \\ 0 & 2 \end{bmatrix}$

a_{21}

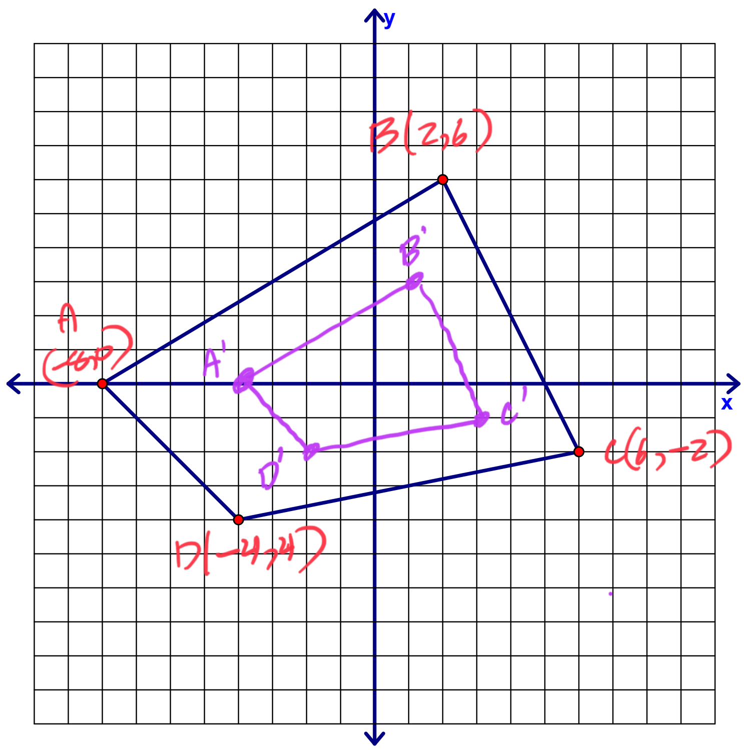
$4(0 - 2x) = -48$
$-8x = -48$
$x = 6$

$4(2y + 6) = 2$
$8y + 24 = 2$
$8y = -22$
$y = \frac{-22}{8} = \frac{-11}{4}$

This problem may look familiar from Geometry….

Perform a **Dilation** of magnitude $\frac{1}{2}$ on the quadrilateral below:

$$\frac{1}{2}\begin{array}{c}\begin{array}{cccc}A & B & C & D\end{array}\\ \begin{bmatrix}-8 & 2 & 6 & -4\\ 0 & 6 & -2 & -4\end{bmatrix}\end{array} \rightarrow \begin{array}{c}\begin{array}{cccc}A' & B' & C' & D'\end{array}\\ \begin{bmatrix}-4 & 1 & 3 & -2\\ 0 & 3 & -1 & -2\end{bmatrix}\end{array}$$



REAL LIFE problem!
Use matrices to organize the following information about condominium fees.

Condo owners must pay yearly fees to cover the cost of maintenance, landscaping, and remodeling.
The fees this year, in the order from above, are \$96, \$18, and \$66 for a 1-bedroom unit,
and \$128, \$24, and \$88 for a 2-bedroom unit.

The fees next year, in the order from above, will be \$105, \$20, and \$73 for a 1-bedroom
and \$141, \$26, and \$97 for a 2-bedroom unit.

Use matrices to organize the information- label your rows and columns!!

This Year

$$\begin{array}{c|ccc} & M & L & R \\ \hline 1\,BR & 96 & 18 & 66 \\ 2\,BR & 128 & 24 & 88 \end{array}$$

A

Next Year

$$\begin{array}{c|ccc} & M & L & R \\ \hline 1\,BR & 105 & 20 & 73 \\ 2\,BR & 141 & 26 & 97 \end{array}$$

B

What are the differences in fees from this year to next year?

$$B - A = \begin{array}{c|ccc} & M & L & R \\ \hline 1\,BR & 9 & 2 & 7 \\ 2\,BR & 13 & 2 & 9 \end{array}$$

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